Let θ be the parameter of a probability distribution. Suppose that we want to test $H_0 : \theta = \theta_0$.

For example, maybe we want to test $H_0 : p = 0.5$ in the binomial setting.

If the MLE estimator of θ is $\hat{\theta}_{\text{MLE}}$, then when the sample size is large, the test statistic

$$Z = \frac{\hat{\theta}_{\text{MLE}} - \text{E}(\hat{\theta}_{\text{MLE}})}{\sqrt{\text{Var}(\hat{\theta}_{\text{MLE}})}},$$

where $\sqrt{\text{Var}(\hat{\theta}_{\text{MLE}})}$ is the standard error of the estimator, has an approximate standard normal distribution. This is equivalent to saying Z^2 has an approximate χ^2 distribution with 1 df.

In the binomial setting,

- $\theta = p$
- $\hat{\theta}_{\text{MLE}} = \hat{p} = \frac{x}{n}$
- $E(\hat{p}) = p$
- $\text{Var}(\hat{p}) = \frac{p(1-p)}{n}$

The problem is that the true value of p is unknown.

(1) If the null value is true we can use the null value of p in the standard error:

$$\text{SE} = \sqrt{\text{Var}(\hat{p})} = \frac{p_0(1-p_0)}{n}.$$

This is the quantity we have traditionally used in the denominator of the Z-test statistic. This is called a *Score Test*.

Definition A *Score Statistic* is one that uses the expression for the standard error evaluated at the null value of the unknown parameter.

Example For the binomial, the test statistic would look like

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}.$$

The Z-test using this test statistic, or the corresponding chi-square test that uses Z^2 is called a *score test*.

This is the standard statistic for a hypothesis test.

(2) However, for a confidence interval, we instead use

$$\text{SE} = \sqrt{\text{Var}(\hat{p})} = \frac{\hat{p}(1-\hat{p})}{n},$$

which is the estimated value of the standard error. It uses the value of the MLE in the expression of the standard error. This is called a *Wald Test*.

Definition A *Wald Statistic* is one that uses the expression for the standard error evaluated at the MLE for the unknown parameter.

Example For the binomial, the test statistic would look like

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}.$$

The hypothesis test using this statistic, or the corresponding chi-square test that uses Z^2 is called a *Wald Test*.

Note: If you take this test statistic and solve for the value of p_0, you get the standard formula for a confidence interval for a proportion.